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History and Principles of Microtonal Keyboards

Introduction

As real-time computer music performance systems become more widespread, the question of controller design becomes increasingly pertinent. The flexibility of pitch afforded by computer technology suggests the use of new input devices optimized for playing in arbitrary tuning systems. In particular, keyboards are well-suited for polyphonic playing, and there is a legacy of historical microtonal keyboards that can serve as models for controller design.

Several motivations for using a microtonal keyboard in computer music can be discerned. The obvious use is for live performance—microtonal music no longer needs to be primarily restricted to tape music on the one hand and to the musician-craftsman who constructs special acoustic instruments on the other. An equally compelling motivation drives the composer of microtonal music. The real-time aural feedback provided by such a device can open the door to experimentation with many tuning systems whose harmonic resources might otherwise remain untapped. A flexible device for real-time pitch control could also be of use in psychoacoustic research.

The greater portion of this article outlines the history of microtonal keyboards, with a view towards establishing the most useful design principles. The final section considers how these principles can be adopted for synthesizer control. A programmable keyboard is particularly useful, allowing a variety of tuning systems and key layouts; software written by the author for such a purpose is described. With such a device, different keyboard layouts can be used to match the tuning system and the nature of the musician's usage, as will be explained.

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Terminology

By "microtonal," I refer primarily to tuning systems with more than twelve notes per octave. However, much of what will be discussed is relevant to nonstandard tunings in general. The somewhat unfortunate prefix "micro-" should not lead the reader to imagine that microtonal compositions are necessarily congested by tiny intervals. A "microtonal keyboard" is any keyboard used to play microtonal music, including retuned standard keyboards, but nonstandard keyboard designs are the principal focus of this study. References to the "black" or "white" keys of a standard keyboard serve only to distinguish between the upper and lower rows of keys, whose actual color varies in some of the examples mentioned.

Although an explication of tuning theory is beyond the scope of this article, some description is necessary in order to understand the motivations for the various designs. *Just intonations* are systems whose pitches are generated by small-integer frequency ratios. For example, the ratios 2/1, 3/2, 4/3, 5/4, and 6/5 correspond to a Justly tuned octave, perfect fifth, perfect fourth, major third, and minor third respectively. Any Just interval is expressible as the product of powers of prime numbers:

$$2^a \times 3^b \times 5^c \times \ldots L^n,$$

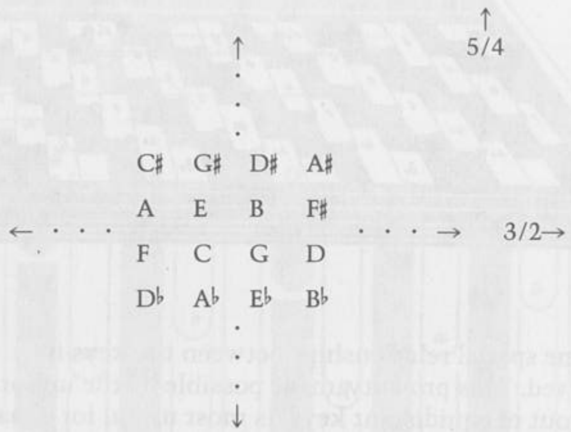
where a through n are integers and L, called the "limit" of the system, is the largest prime number in the series. As an example, a seven-limit Just intonation might include the ratio 14/9, which reduces to

$$2^1 \times 3^{(-2)} \times 5^0 \times 7^1.$$

Figure 1 is a representation of five-limit Just intonation. Horizontally adjacent pitches are a perfect fifth (3/2) apart; the vertical separation is a major third (5/4). A dimension for the 2/1 ratio is not included; it is assumed that each pitch represents its equivalents in all other octaves.

It will be observed that the notes in the top row are not exactly an octave higher than their *en-*

Fig. 1. Five-limit Just intonation matrix.



harmonic equivalents* in the bottom row, because $(5/4)^3 = 125/64 \neq 2/1$. In general, no "wraparound" is possible in such systems, which extend infinitely in all directions with no duplication of pitches. In practice, a finite area of the plane is chosen. Systems with higher limits can be conceptualized as n-dimensional spaces, where the limit is the nth number in the prime number series 3, 5, 7, 11, . . .

Temperaments are systems, some or all of whose intervals correspond to irrational frequency ratios. An *n-tone equal temperament* divides the octave into n equal parts, each of which is expressible as the frequency ratio $2^{(1/n)}/1$. In general, the attraction of Just intonation for microtonal composers is the belief that simple ratios produce better-tuned consonances, whereas the attraction of equal temperaments is that transposition to any degree of the system is possible. The proponents of microtonal keyboards have sought purity of intonation, or a new musical idiom with expanded pitch resources, or both.

Historical Overview of Microtonal Keyboard Design

Keyboards before the Nineteenth Century: The Process of Accretion

In tracing the development of microtonal keyboards, it is instructive to first understand the origin of the standard keyboard with seven white and five black keys per octave. Throughout most of the Middle Ages, keyboards usually had only the diatonic keys, B-flat being the first addition. In the fourteenth century the remaining chromatic degrees were added. The familiar layout with the five chromatic degrees in a higher row appeared on the Halberstadt organ, built in 1361. This arrangement soon became standard. Thus our present-day keyboard evolved from a seven-note modal keyboard. The chromatic keys were added without violating the spatial pattern of the existing keys, a process that I shall refer to as "accretion."

Keyboards with more than twelve pitches per octave were occasionally used in the sixteenth century and later. Major and minor thirds and their inversions had come to be considered consonances, and they were ideally to be tuned in Just intonation. (A practical compromise known as *quarter-comma meantone temperament* allowed the most important ones to be so tuned.) As we saw in Fig. 1, pairs of notes such as G-sharp and A-flat differ by significant amounts in such systems. It was not an uncommon practice to split one or more of the black keys into a front and a rear portion, in order to obtain these enharmonic distinctions.

Enharmonic keyboard designs were carried further by sixteenth- and seventeenth-century theorists and composers including Gioseffo Zarlino (1558), Nicola Vicentino (1555), Francisco Salinas (1577), and Marin Mersenne (1637). One of the most important instruments was Vicentino's "Archicembalo," whose tuning was apparently very close to 31-tone equal temperament (Kaufmann 1970). In addition to splitting the black keys, these designs often called for split white keys (Fig. 2), additional black keys (Fig. 3), or extra manuals. Note that these techniques of splitting keys and inserting new ones, without disrupting the established layout, represent a continuation of the same process of accretion that led to the standard keyboard.

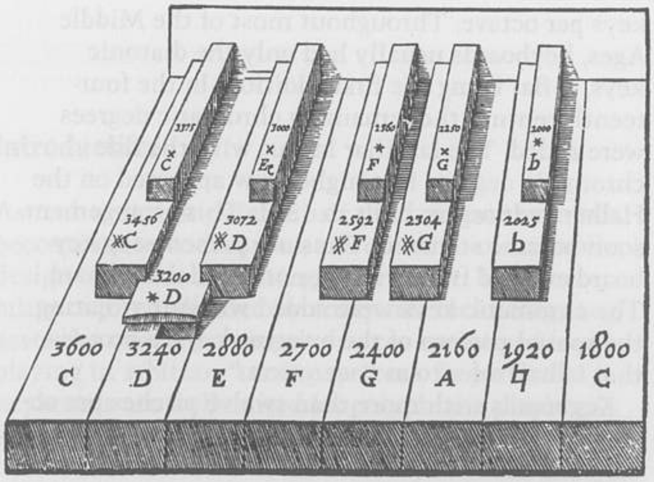
The Nineteenth Century: Bosanquet and Transpositional Invariance

Following relatively little activity in the eighteenth century, important advances in keyboard design

Fig. 2. Harpsichord by Joan Albert Ban (1639), after a design by Mersenne.

Fig. 3. Harpsichord with 31 tones per octave, built by Vito Trasuntino (1606).

Fig. 4. Just intonation harmonium of Colin Brown (c. 1875).



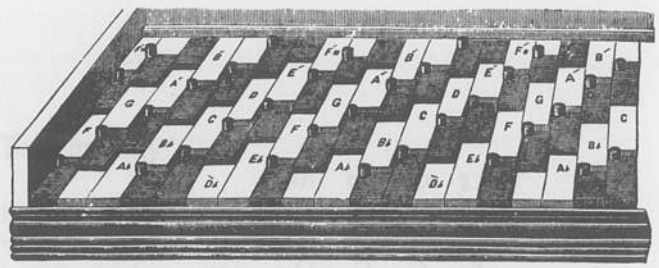
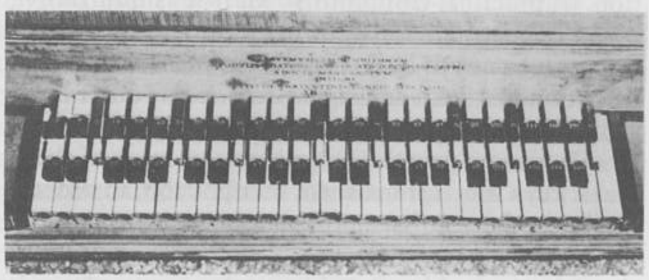


Fig. 3



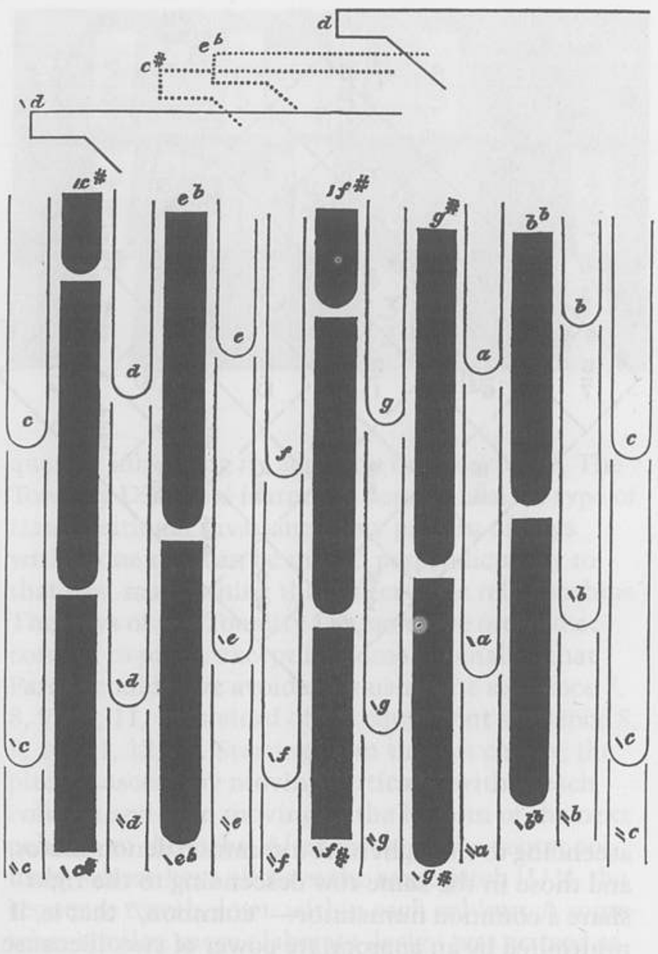
were made in the second half of the last century. This was the age of Helmholtz, whose monumental achievements in musical acoustics lent credence to his espousal of Just intonation. Helmholtz's Justly tuned harmonium was only one of many in this period (Davies 1984), among which Colin Brown's (Fig. 4) was a notable example (Helmholtz 1954).

The design that has had the most lasting impact is the "generalized keyboard" of R. H. M. Bosanquet. Figure 5 shows his harmonium tuned in 53-tone equal temperament, a division of the octave that produces some extremely good approximations of Just intervals. The generalized keyboard has two very useful and related properties: *transpositional invariance* and adaptability to a variety of tuning systems (as its name suggests).

Transpositional invariance allows one to move chords or musical passages to any pitch level while maintaining exactly the same fingering and the same spatial relationships between the keys involved. This property, made possible by the uniform layout of equidistant keys, is most useful for equal temperaments, in which exact transposition to all degrees is possible. The principle of transpositional invariance had been noted long before with reference to nonmicrotonal twelve-note keyboard designs. As early as 1708, Konrad Henfling had proposed a keyboard laid out as two whole-tone scales, and others have improved the design, which is much more logical for twelve-tone equal temperament than the traditional keyboard—and probably easier to learn to play (Reed 1973). Bosanquet applied the principle of transpositional invariance to the microtonal realm. What makes his design particularly successful is that the traditional keyboard pattern is retained within it (Fig. 5), although less transparently than in accretion-based designs. Despite the initially formidable appearance of this instrument, the logic of its design simplifies playing. The whole-tone rows rise upward as they move to the right and to the back. As a result, all major scales can be fingered like A major on a standard keyboard (although this can vary depending upon the tuning), with similar consistency for other melodic patterns and chords.

Bosanquet's keyboard was designed for what he termed *regular cyclic temperaments*, namely, those equal temperaments whose pitches form a single cycle of close-to-perfect fifths (Bosanquet 1875). The popular 19-tone and 31-tone equal temperaments are members of this category. It is apparent that systems such as 24-tone equal temperament have more than one cycle of fifths (two cycles a quarter tone apart in this instance), and that other systems such as 13-tone equal temperament do not

Fig. 5. R. H. M. Bosanquet's enharmonic harmonium (1872–3), with generalized keyboard. Top: cross section of four keys. Bottom: Front view of part of the keyboard. Whole tones move upward to the right; major scales fall in a horizontal line; vertically adjacent notes form the smallest interval in the tuning system. Any chord or melodic pattern is fingered the same in all transpositions.



have a close approximation to the perfect fifth. Such systems do not fit well onto Bosanquet's generalized keyboard pattern in that their pitches do not all fall onto the locations predicted by the pattern of white and black keys. However, like all equal temperaments, they do fit in such a way as to exhibit transpositional invariance. A generalized keyboard can also be used for non-equal-tempered systems such as Just intonation or non-Western scales, as demonstrated by Wilson (1974) (Fig. 6).

The Early Twentieth Century: Quarter-tone Keyboards

In the years from the turn of the century to World War II, many musicians were interested in 24-tone equal temperament, which provides microtones while retaining the familiar twelve-tone temperament as a subset. In 1892 G. A. Behrens-Senegalden built his "achromatisches Klavier"; this quarter-tone instrument was followed over the next four decades by a good number of other quarter-tone pianos and harmoniums. A number of designs were used. Most followed the obvious plan of duplicating the standard twelve-note keyboard and tuning the two manuals a quarter tone apart. To simplify fingering, a number of the designers added a third manual in the back, tuned the same as the front one; the keys in some of the rows were often shortened as well. The keyboards of Max Meyer (Fig. 7) and Willi von Möllendorff, in contrast, were based upon the accretion principle. Rather than duplicating the original manual, they interspersed the quarter-tone keys among the normal twelve. There were some instances of quarter-tone keyboards later in the century, but they are outnumbered by other microtonal systems.

Some early twentieth-century composers were interested in other divisions of the whole tone: Ferruccio Busoni had a third-tone harmonium and commissioned one in sixth tones, and Alois Hába likewise had a sixth-tone harmonium constructed, in addition to some quarter-tone pianos (Hába 1971). The Mexican composers Julian Carrillo and Augusto Novaro had special microtonal pianos constructed with normal keyboards. Carrillo's set of pianos covered all the equal temperaments in the series 18, 24, 30, 36, . . . , 96. (The last of these had 97 keys, for a total span of one octave!) Keyboards in 19-tone equal temperament also received some attention in the first quarter of the century (Mandelbaum 1961).

Partch and Just Intonation Matrices

Experimentation with Just intonation keyboards continued during the early twentieth century alongside the quarter-tone efforts. One of these was Thaddeus Cahill's Telharmonium, a massive predecessor of the synthesizer that in its second version (1906) included Just intonation with up to 36 pitches per octave (Pierce 1924).

Fig. 6. Just intonation layout for the Wilson-Hackleman clavichord (1976).

Fig. 7. Quarter-tone harmonium of Max Meyer (1902).

Fig. 8. Partch's eleven-limit "Tonality Diamond." The ratio for each key is formed by the product of the numbers above the diamond whose rows intersect at that key. Ratios are normalized by powers of 2 in order to fall in the range (1/1, 2/1).

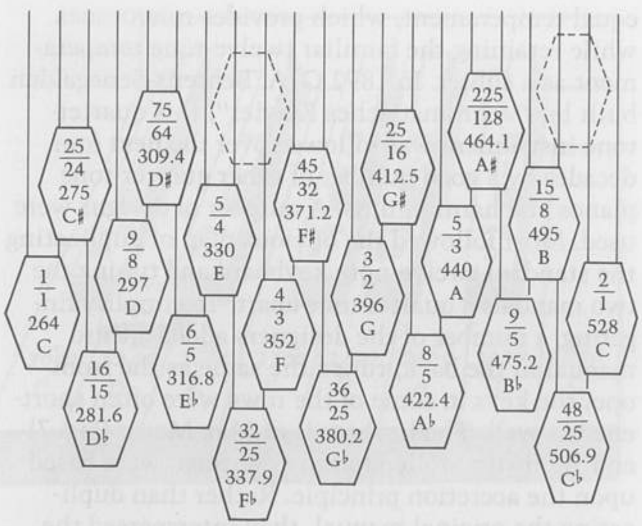
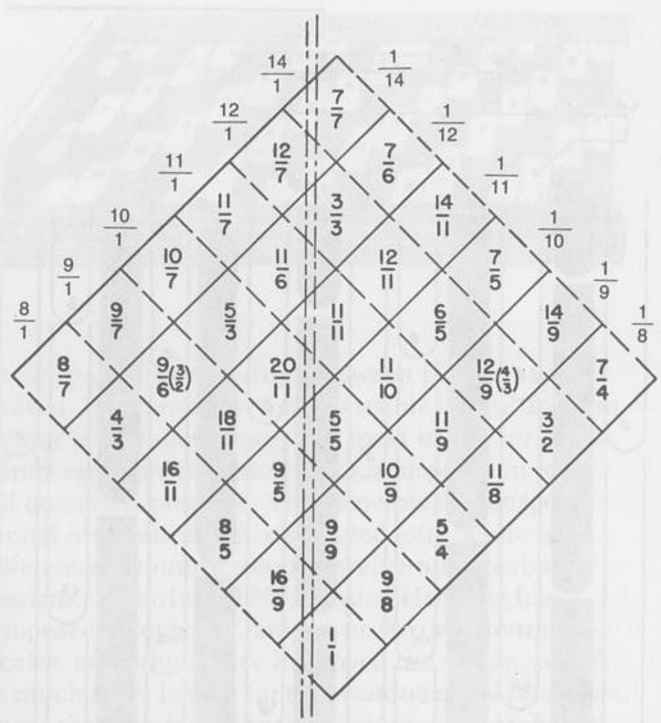


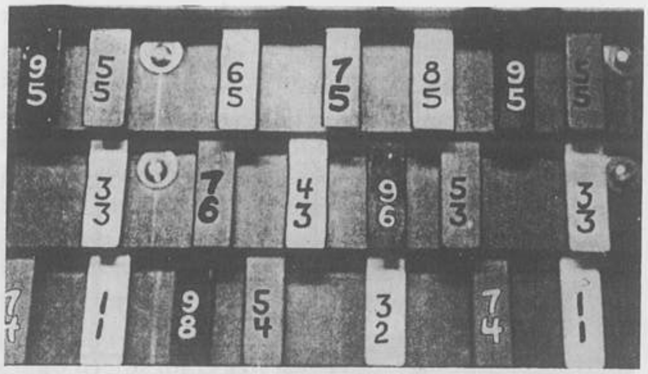
Fig. 7



One particularly innovative design was Harry Partch's "Ptolemy" keyboard (c. 1935), an illustration of which is given in the first edition of *The Genesis of a Music* (Partch 1949). (Partch might have been inspired by his reading of Meyer [1929], who depicts a similar diamond on page 22, though not with reference to keyboard design.) The main keyboard of this instrument was similar to a typewriter, but with somewhat larger, circular keys. Above the main keyboard was a smaller keypad arranged in a kind of matrix that Partch called a "Tonality Diamond" (Fig. 8). Each key in the Tonality Diamond is represented by its frequency ratio to a reference pitch. Keys in the same diagonal row ascending to the right have a common denominator, and those in the same row descending to the right share a common numerator—"common," that is, if multiplied by an appropriate power of two. (Because he used ratios in place of pitch names, Partch's nomenclature "fudges" by powers of two so that each note can have the same name in every octave.) The Tonality Diamond has been used in an extended form on a harpsichord built in 1974 by Norman Henry, who is currently building a piano based on the same principle. Like Partch, Henry uses color coding (different colors corresponding to different numerators and denominators) to help orient the player (Henry 1985).

With Just intonation designs, it is difficult to reconcile two competing features that are desirable for a keyboard: transpositional invariance and ordering of pitches along the horizontal axis. It will be observed that the Just intonation matrix of Fig. 1 exhibits complete transpositional invariance; unfortunately the pitches do not lie in an ordered se-

Fig. 9. The "Referential Organ" of Boomsliter and Creel (c. 1961). (Courtesy of Paul Boomsliter.)

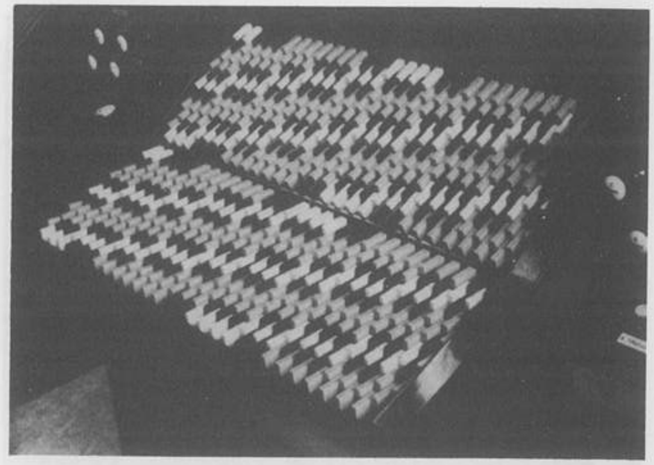


quence, inhibiting its adoption on a keyboard. The Tonality Diamond features a certain limited type of transpositional invariance: any pattern of keys within one row can be moved perpendicularly to that row, maintaining their intervallic relationships. The keys of the Tonality Diamond are ordered according to pitch, except for some anomalies that Partch could have avoided by using the sequence 7, 8, 9, 10, 11, 12 instead of his equivalent sequence 8, 9, 10, 11, 12, 14. Starting from the left corner, the pitches ascend by moving vertically within each column and then moving to the bottom of the next column to the right. After reaching the center column, whose keys all have the same pitch (1/1), the sequence travels down within each column. A somewhat similar, more elaborate design was arrived at independently by Mycielski (1972). Both these layouts associate numerically (and acoustically) related pitches with each other. The "Referential Organ" of Boomsliter and Creel (1962) retains some of these associations while keeping the horizontal displacement proportional to pitch (Fig. 9).

Other Nonstandard Keyboard Designs in the Twentieth Century

Bosanquet's generalized keyboard has continued to exert considerable influence on microtonal design in the twentieth century, ranging from plain imitations to various modifications (von Oettingen 1916; Fickénscher 1941; Schafer and Piehl 1947; Wilson 1974; Fokker 1975; Secor 1975). Perhaps the best-

Fig. 10. The organ of A. D. Fokker, in 31-tone equal temperament (1950). (Courtesy of the Verlag für systematische Musikwissenschaft.)



known of these is the organ built for Adriaan Fokker in 1950, which is tuned in 31-tone equal temperament (Fig. 10). In the 1970s the Archifoon, an electronic offshoot of this organ, was built; Robert Moog also built two synthesizers with generalized keyboards.

A generalized keyboard was developed for the Motorola Scalatron (Fig. 11), which was tunable to different microtonal scales, but whose commercial production in the 1970s was short-lived (Secor 1975). Whereas most of the earlier designs retain Bosanquet's idea of long, narrow, overlapping keys, the Archifoon has shorter rectangular keys, Secor's Scalatron has oval keys, and Wilson's clavichord keyboard uses hexagonal ones. These modifications reduce key length as a tradeoff for a wider key surface.

Besides the generalized keyboard, other concepts have recurred in modern designs, and some new ideas have cropped up. The principle of accretion continues to be used (Yunik and Swift 1980, 1982; Conviser 1980; Daniélou 1978). Alain Daniélou has carried the principle so far as to segment the standard keyboard into enough pieces for a 53-tone scale (Fig. 12). An interesting "Beehive Keyboard" design by Jim Davis uses a tempered adaptation of the Just intonation matrix (Davis 1986). Another phenomenon in the twentieth century has been designs that separate microtonal scales into new groupings—analogous to the traditional grouping of white and

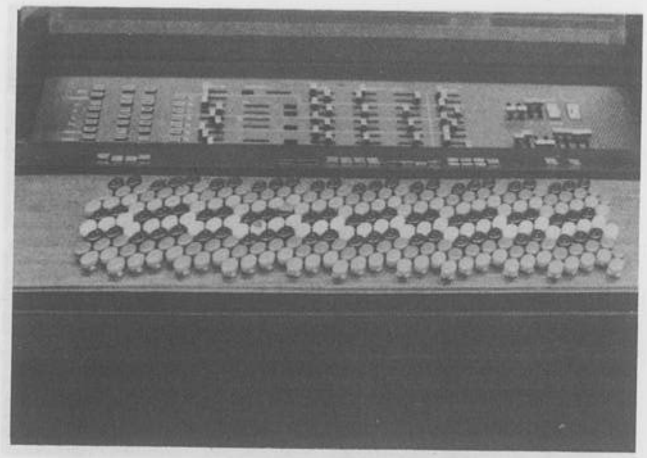


Fig. 11. The Motorola Scalatron with a generalized keyboard (1975).

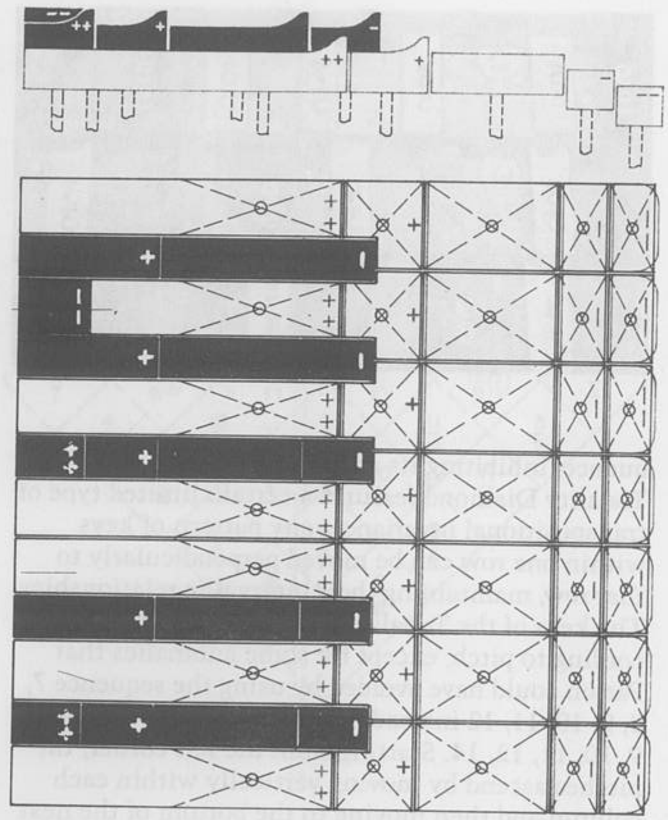


Fig. 12. Cross section and top view of Alain Daniélou's 53-note octave (1967). "+" and "−" refer to alterations by about twenty cents. (From Daniélou 1978.)

Fig. 13

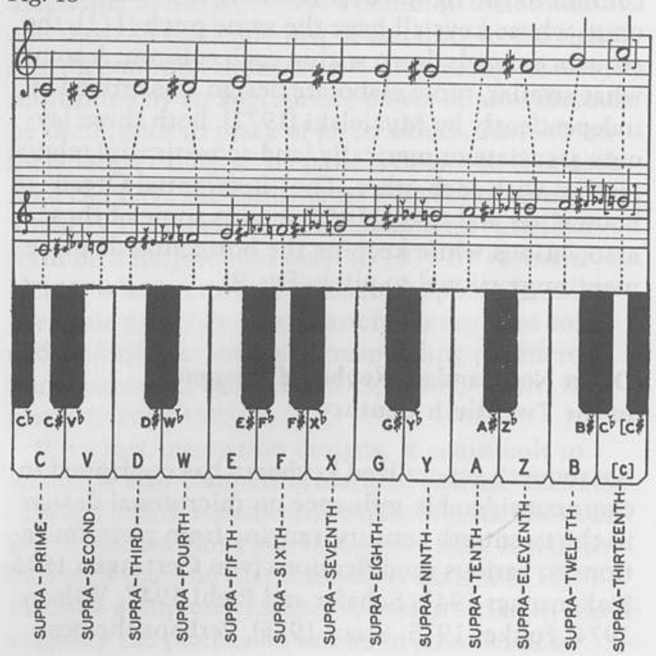


Fig. 13. Joseph Yasser's division of a 19-note scale into two groups (1932).

black keys—in order to highlight some theoretical feature of the tuning system (Fig. 13) (Yasser 1932). Along similar lines, John Chalmers has suggested layouts embodying the groupings for n-tone equal temperaments formalized by him and Ervin Wilson (Chalmers 1980). Note that irregular groupings conflict with the requirements of transpositional invariance, as on the standard keyboard. To avoid this conflict, the groupings could instead be made explicit by appropriate color coding on a generalized keyboard.

The Use of Standard Keyboards

The discussion thus far has concentrated on nonstandard key layouts, but there are a number of cases where standard keyboards were used on instruments with microtonal capabilities. Let us analyze this approach, since it is a convenient one for computer music.

Keyboards for Microtonal Computer Music

In the past, the necessity for specially constructed instruments thwarted the systematic use of new tuning systems. The experimenters of other centuries would no doubt be highly envious of the ease that computers bring to the exploration of

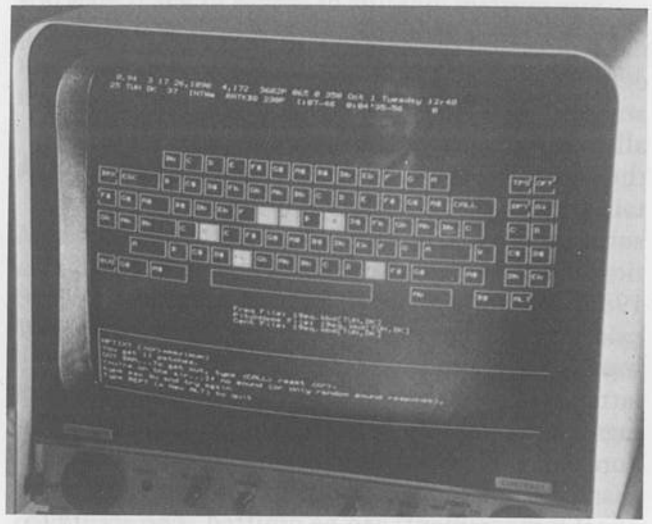


Fig. 14. Screen image produced by a computer program by the author for simulating a microtonal keyboard (1985).

new pitch resources. Whereas many of the old instruments had to be dedicated to one particular microtonal scale, a comparatively simple piece of software can instantly put any tuning system imaginable at the fingertips of the computer musician. Digital synthesizers are perfectly capable of producing microtonal music without the use of keyboards, of course. But the ability to polyphonically play an arbitrary set of pitches in real time can greatly facilitate the process of learning the features of a tuning system. If even great composers have often found a piano useful for trying out their musical ideas in a conventional tuning, how much more useful would such real-time aural feedback be in an unfamiliar intonational terrain?

To provide an initial tool for such exploration, I have created a computer program at Stanford's Center for Computer Research in Music and Acoustics (CCRMA) that allows the user to specify arbitrary tuning systems and play with them in real time (Keislar 1986a, 1986b). This program makes use of earlier code by Bill Schottstaedt and David Jaffe for real-time control of CCRMA's Systems Concepts Digital Synthesizer ("Samson Box"). The computer keyboard is used as the controller—a simple but also appropriate choice, considering the basic similarity of a typewriter keyboard to some of the generalized microtonal keyboards that have been constructed in the past.

If each pitch is to have its own key, one can use multiple manuals. This technique was common with quarter-tone instruments, and continues to be employed (Vogel 1975; Richter-Herf 1975). The only alternative is to spread the musical octave out over more than a physical octave on the keyboard, as Carrillo did. Although a main attraction of using the standard keyboard is the ability to capitalize on existing performance skills, both of these solutions do introduce new technical problems: playing notes on two manuals simultaneously with one hand requires a new fingering technique, and Carrillo's single-manual approach reduces the performer's effective hand span. The latter approach also means that a given pitch will tend to have a different appearance in each octave, so that the keyboard pattern no longer provides clues as to the resultant pitch. This is less problematic for performance from a strictly notated score (since the notation can correspond to the physical keyboard instead of to the sound) than for improvisational playing, where anticipation of the sounding pitch is necessary.

Alternately, a single key can serve duty for different pitches at different times. Combining a standard keyboard with secondary performance devices such as pedals that select different sets of pitches was a technique used already in the eighteenth century. The concept has been used in the twentieth century as well. Automatic adjustment of pitch has also been used to realize Just intonation on standard keyboards (Groven 1955; Darreg 1981; Ganter, Henkel, and Wille 1985; Waage 1985). For the performer, such systems are convenient in that no new playing techniques are required: what is relinquished is the ability to select arbitrary sets of pitches from a microtonal scale.

A picture of the computer keyboard is presented on the screen, with each key labelled according to its pitch (Fig. 14). As each physical key is depressed, the corresponding key on the display lights up, allowing one to easily keep track of one's place on the keyboard; this graphic feedback greatly simplifies the process of learning to play in a variety of tuning systems and key layouts. The label on each of the keys on the screen can display the note name with an optional octave number, its frequency expressed in hertz, in cents, as a ratio, or by its original typewriter-key name. There are control keys for octave displacement, transposition by arbitrary amounts, and calculation of the intervals between all simultaneously depressed keys, as well as for the activation of such functions as a simulated sustain pedal and a sequencer routine that records the subsequent performance as code in the compositional programming language Pla (Schottstaedt 1983). More importantly, one can either use preexisting pitch layouts or design one's own by specifying the note name and tuning of every key. The latter option is simplified by certain shortcuts; tunings can be input in cents, ratios, or hertz, and redundant information (such as frequencies for more than octave, or indeed for more than one note in an equal temperament) can be omitted. The ability to experiment with different layouts allows one to approximate various historical designs, and to develop concepts that might be applied to the construction of special microtonal keyboards.

For maximum flexibility, one would like a keyboard with programmable key configuration, so that keys could have different sizes and shapes from typewriter keys. Johnstone (1985) has described the Rolky, a system with a touch plate capable of responding to multiple contact areas. Any sort of key layout can be graphically projected onto the touch plate. Pressure sensitivity is simulated by responding to the contact area of the finger on the glass. The ideal microtonal controller would be similarly configurable but three-dimensional, with movable keys of variable size and shape, in order to maximize tactile feedback. Different forms of sensitivity (velocity, pressure, displacement in up to three dimensions) could be employed. Imagine a flat surface divided into many tiny squares, each the top of a piston that could be moved up or down like a piano key. A "key" would be an area, drawn by the user, that contained many of these pistons. All the pistons would move together whenever any of them were pressed, giving the same effect as a single solid key. A three-dimensional, programmable control panel such as this could have many extramusical applications as well.

Certainly, there are other possible modes in which microtonal keyboards can be integrated into a computer system. One can imagine a spectrum ranging from complete performer control to complete composer (or program) control. At one extreme is the conventional approach of providing a separate key for each pitch, such that the performer can simultaneously access any set of notes in the tuning system. As we have noted, the key layout can be simplified, at the expense of freedom of selection, with supplementary performance controls for reassigning the pitches of the keys, or with automatic assignment. Control of pitch can be removed entirely from the performer—for example, if the composer's note list is stored in memory. Max Mathews has introduced the notion of a keyboard with only ten keys—one for each finger—for this performance mode. And, of course, a single key depression can trigger events much more complex than a single note.

Polyphonic pitchbend is a useful feature that was less practical before the invention of electronics (with the possible exception of the clavichord). Snell (1983) has proposed the use of a keyboard that has the standard layout, but with the black keys sloping down at the rear to the same plane as the whites. This flat rear portion of the keyboard would function as a ribbon controller that allowed precise, independent pitchbend on each note of a polyphonic texture. With a flat surface, such as that of the Rolky, one could likewise create a generalized keyboard pattern in which the spaces between the keys could be used to bend from one pitch to another.

No discussion of keyboard control in computer music would be complete without reference to the Musical Instrument Digital Interface (MIDI). As of this writing, MIDI does not support nonstandard tunings, although at least one manufacturer has marketed an inexpensive synthesis unit (the Yamaha FB01) that can generate microtones by responding to system-exclusive fractional note numbers. For most synthesizers, however, one must resort to the "kludge" of routing keyboard performance data via a computer that adds appropriate pitchbend information to each note. Because pitchbend is global to all notes within a MIDI channel, only one note per channel can be played at a time, wasting much of the synthesizer's capability. It is to be hoped that manufacturers will agree upon a standard method for tuning individual notes.

Conclusions

What, then, would be the ideal microtonal keyboard for computer music? The answer to this question depends on the use to which the keyboard is put. For situations where the employment of familiar keyboard technique is of higher priority than a logical key layout, as for example in virtuosic performance of strictly notated music, the use of one or more standard manuals is a likely solution. In instances where the musician is involved in making decisions about the selection of pitch (for example, in improvisation, or when studying the features of a tuning system), a nonstandard keyboard design may be preferable, particularly when it offers new benefits such as transpositional invariance. The use of such techniques as video display or color coding can help solve the problem of initial disorientation.

We have gleaned several principles from the history of microtonal keyboards that can be useful to consider when deciding upon a nonstandard key layout. The popularity of designs based on the accretion principle stresses the importance of retaining familiar patterns where possible, and the large number of early instruments with split keys shows that accretion-based designs are viable for systems with not many more than twelve notes per octave. However, given that the traditional keyboard itself has a suboptimal design created by a process of accretion, it is clear that microtonal designs building upon this pattern will suffer from the same shortcomings—and probably even more severely. For tuning systems with many notes per octave, or for those in which extensive transposition is possible, an equidistant key layout such as the Bosanquet generalized keyboard is definitely preferable. In cases where the user plans to explore only one specific tuning system, a special-purpose keyboard highlighting properties of that system may be more desirable; the "Tonality Diamond" of Harry Partch is one example.

Computer technology has greatly increased our ability to experiment with all these tunings and key layouts. Programmable, two-dimensional surfaces allow the creation of arbitrary key layouts, and in the future, even three-dimensional sensors will probably become physically configurable by software. Lack of access to suitable instruments has always been an albatross around the neck of microtonal music; finally, the resources of different tuning systems are becoming easily available to all.

Acknowledgments

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