# A novel thick-film strain transducer using piezoelectric paint

**J M Hale*** and **J Tuck**
Department of Mechanical, Materials and Manufacturing Engineering, University of Newcastle upon Tyne, UK

**Abstract:** This paper describes the manufacture and testing of a novel strain transducer for use in structural vibration monitoring, based on piezoelectric paint. The materials selection and development of application techniques are outlined for the three principal components of the transducer (paint binder, piezoelectric material and electrode). The final design comprised a suspension of milled lead zirconate/lead titanate (PZT) ceramic powder in lacquer with a gold electrode formed by vapour deposition. The transducers were tested for variability and for dynamic properties (bandwidth and dynamic range). It was found that there is considerable scatter between the output sensitivities of individual transducers, but that the dynamic properties are very consistent. It is shown that the techniques developed form the basis for a viable vibration transducer in the range 20–600 Hz.

**Keywords:** piezoelectric paint, thick film transducer, sensor

## NOTATION

| | |
|---|---|
| $a$ | distance from the beam neutral axis |
| $A, B, C, D$ | constants defining the vibration mode shape |
| $E$ | Young's modulus |
| $I$ | beam second moment of area |
| $K$ | the constant value $1/2A\beta^2t$ |
| $l$ | beam length |
| $M$ | bending moment |
| $t$ | beam thickness |
| $T$ | kinetic energy |
| $U$ | strain energy |
| $x$ | position along the beam |
| $y$ | displacement normal to the beam axis |
| $Y$ | beam tip deflection |
| $\beta$ | value depending on the vibration mode and beam length |
| $\varepsilon$ | strain |
| $\rho$ | beam mass per unit length |
| $\sigma$ | stress |
| $\omega$ | vibration frequency (rad/s) |

## 1 INTRODUCTION

Piezoelectric materials have many potential applications in strain measurement. However, a serious limitation on these applications is the difficulty of attaching the piezoelectric material to the surface to be tested. A solid ceramic cannot be applied easily to a curved surface, and even a polymer film cannot be applied to a double curved profile or any irregular surface such as a corner or weld.

Thick film piezoelectric transducers have been made by the conventional thick film process using special inks loaded with piezoelectric material in suspension [**1**]. The drawback is the need for screen printing to apply the ink and high-temperature firing to cure it, which together limit the size and shape of the body to which the transducer can be applied. It also makes the process unsuitable for use on temperature sensitive materials such as plastics and FRP composites.

It has been proposed by Egusa and Iwasawa [**2**] that it would be easier to apply the ceramic in the form of a paint. This would conform to the surface and adhere well even in the most inaccessible places. Some promising experiments are also reported, in which the mode shape of a vibrating beam is measured.

More recently the technique has been used to make much higher-frequency sensors using ceramic piezoelectric paints. Egusa and Iwasawa [**3**] describe acoustic emission sensors for use up to 1.2 MHz and Lukacs *et al.* [**4**] describe ultrasonic transducers in the range 80–200 MHz, both using piezoelectric paints based on lead zirconate/lead titanate (PZT) ceramic.

*The MS was received on 29 October 1997 and was accepted after revision for publication on 1 September 1998.*

* *Corresponding author: Department of Mechanical, Materials and Manufacturing Engineering, Stephenson Building, University of Newcastle upon Tyne, Newcastle upon Tyne NE1 7RU, UK.*

The main problem with this work concerns the application of the transducer material. The piezoelectric paint was made simply by mixing PZT powder into epoxy resin, which could only be applied by an inconvenient scraping technique. In addition, the conducting electrode that has to be applied to the surface of the paint was made by screen printing a conducting ink, which negates the advantage of avoiding the screen printing process for the piezoelectric film. The work described here seeks to avoid these problems by developing a paint-based thick film piezoelectric transducer that can be applied easily and would be suitable for uneven surfaces and temperature sensitive substrate materials.

## 2 CONCEPT

All piezoelectric strain sensors are based on the same principle. Piezoelectric materials are made up of elemental dipoles which are aligned by some mechanical or electrical process to form a polarized sensor element. When the material is strained in the alignment direction the elemental dipoles are also strained, thus altering the dipole moment (i.e. the effective distance between the poles) of each and thus changing the potential difference across the sensor element. A conducting layer is provided on the opposing faces of the sensor element so that a charge may gather and drive a measuring instrument connected across the faces, either directly or via a charge amplifier.

Paints come in many forms and have generated a technology of their own [**5**]. However, they all have two basic components: a liquid base or binder containing a suspension of solid particles (the pigment). The novel idea, attributed by Egusa and Iwasawa [**2**] to Hanner *et al.* [**6**], was to make a paint with piezoelectric properties by replacing the pigment material with a fine powder of piezoelectric material. When the paint dries, the piezoelectric particles are locked in position and so form a solid piezoelectric composite film. If the material is then treated to align the particles and electrodes are provided on each side to allow measurement of the voltage generated under strain, this is potentially a thick film strain transducer.

The techniques described below, based on the work of Egusa and Iwasawa [**2**], produce a transducer that measures strain in the direction normal to the plane of the film, i.e. the strain due to Poisson's ratio rather than the true surface strain of the body on which the transducer is mounted. This orientation is used for ease of manufacture and has two effects:

1. The measured strain is only of the order of one third of the surface strain, so the output of the transducer is smaller than it might have been. However, as shown below, it is still substantial.
2. The transducer is non-directional. This could be seen as either a strength or a weakness, but certainly has the advantage of eliminating the problem of accurate alignment traditionally associated with mounting strain gauges.

The objective of the present work was to establish the viability of the piezoelectric paint concept by making transducers in small batch quantities and testing their performance at the moderate frequencies required for structural vibration work. The first task was to select suitable materials and develop suitable techniques for applying them. These two objectives were intimately bound together. The piezoelectric material and the binder had to combine to form a paint that would be easy to apply and would cure to form a uniform layer. Similarly, the conducting layer had to be suitable for application on to the paint surface without interfering with any of the properties of the paint or the curing (drying) process.

The next task was to develop a process by which the piezoelectric particles may be aligned reliably and permanently. The process employed depends on the type of piezoelectric material used, but for the ceramic material selected (see below) the conventional technique is electrical poling [**7**], by which the material is exposed to a strong constant electric field for a prolonged period. The intensity of the electric field and the duration of application were determined experimentally.

The properties of the transducers thus made were then tested. A large number of specimens was tested to determine quickly the effect of piezoelectric powder concentration in the paint and the variability between nominally identical transducers.

Finally, a thorough test of the dynamic response of three representative transducers was made. The dynamic range and bandwidth of these three transducers were measured under steady state vibration, using a strain gauge mounted on the test specimen as a reference.

## 3 MATERIAL SELECTION AND APPLICATION TECHNIQUES

A paint-based piezoelectric sensor requires three main components: piezoelectric material in fine powder form, a resin base (also known as a 'binder' or 'vehicle') to carry the powder in suspension during application and bind it together on curing, and an electrically conducting surface layer. For most engineering applications the substrate (the surface of the body whose strain is to be measured) will be metallic and so it is not necessary to provide an independent conducting layer underneath the paint.

For a paint-based sensor to work effectively there are two requirements of the piezoelectric component. It must have a high piezoelectric constant and the particles must be capable of dense packing. Both points are necessary for a large potential difference to be generated at moderate strains. It was thus necessary to identify a suitable piezoelectric material that could be ground to a fine powder.

The attributes required of the resin matrix are that it should flow readily with a high proportion of powder in suspension in the uncured state to give an easy application and an even covering with a smooth surface, that it should cure to a stiff solid with minimal voiding to transmit strain into the piezoelectric particles, and that it should be a dielectric (a material of high electrical resistivity) to isolate the individual piezoelectric particles and prevent a short circuit through the material.

The other material to be selected for the sensor was the conductive surface layer. This had to be light and flexible to add negligible mass and stiffness to the system, very low resistance and capable of conforming to any double-curvature surface on which the paint was laid.

### 3.1 Piezoelectric material selection

The effectiveness of a piezoelectric transducer is clearly heavily dependent on the piezoelectric material used. There is a wide range of materials that exhibit piezoelectric properties to a greater or lesser extent [**7**], but the most powerful by far are the lead zirconate/lead titanate (PZT) ceramics.

Transducer grades of PZT are readily available in powder form and an appropriate grade was selected with advice from a commercial supplier, Morgan Matroc Ltd. This is a milled sintering powder consisting of fairly uniform particles of 1–2 μm (0.001–0.002 mm) diameter and approximately spherical form. The powder thus appeared well suited to the application and was used in the subsequent work.

### 3.2 Resin ‘binder’ selection and development of paint application technique

Selection of a suitable resin to act as a binder for the paint formed a significant part of the work. It was necessary to identify a material that would hold a high fraction of the PZT powder in suspension but still flow freely during application, would cure quickly at room temperature, would prevent significant settling out of the powder during the curing process and would dry to a smooth, even finish.

Identifying a suitable resin binder was a three-stage process. Firstly, a range of commercial paints and resins was tried for ease of mixing with high concentrations of PZT powder and applying by brush and spray. Eventually a lacquer was found that was easy to apply, even with powder concentrations of up to 80 per cent by weight. This lacquer, marketed by LeFranc and Bourgeois as a ‘colourless painting medium’ and widely available from artists’ suppliers, was adopted and taken forward.

Secondly, application trials by brush and spray were undertaken on paints with various concentrations of PZT in the lacquer. It was found that paints with up to 75 per cent PZT could be sprayed, giving a blemish-free, smooth finish. Paints of the highest PZT concentrations of up to 80 per cent could only be applied by brush, gave an uneven surface and required great care to avoid occasional pinholes that could allow the electrode layer to short circuit through the piezoelectric layer to the substrate.

Finally, the cured paint was analysed by scanning electron microscopy (SEM). Paint layers of various PZT concentrations were made by spraying on to an aluminium substrate from which cross-sections were cut, set into epoxy and polished to form SEM specimens. It was clear from this analysis that for all PZT concentrations the particles were distributed evenly through the matrix and there was no observed tendency for them to settle out during the early stages of the cure. The lacquer was thus adopted as the binder for the paint to be taken forward for functional trials as a sensor.

### 3.3 Electrode material selection and development of deposition technique

For most engineering applications, the substrate (the material of the body whose strain is to be measured) is metallic and so acts as the bottom electrode. However, the top electrode is critical for the transducer. There are two separate functions of the electrodes: to provide a pickup for the finished transducer and to facilitate the high voltage poling process that aligns the piezoelectric dipoles.

In previous work [**2**] the electrode was produced by applying conductive ink on to the paint by screen printing. This approach was not followed here because of the perceived problems of printing on to complex curved surfaces, which is seen as an important application. Six materials were evaluated for the electrode, of which only one, vapour deposited gold, was found to be entirely satisfactory.

Brass shim and aluminium foil applied to the wet paint were found to be unsuccessful because they prevented the paint from drying properly and left a layer of moisture between the surfaces, either contaminants or excess liquid from the cure which is prevented from evaporating by the barrier presented by the solid metal electrode. Cardiac monitor patches were found to be quite effective and were used in some early work. These are conductive plastic foam pads approximately 3 mm thick and supplied with an adhesive film on one side. They are intended for use in medical applications to pick up the small electrical signals at the surface of the skin generated by a patient’s heart and other muscular activity.

The patches were stuck directly to the dry paint layer and were found to give generally good results, both during the high-voltage poling process and in use as a pickup for the transducer. However, the cardiac patches were cumbersome to use and added significantly to the mass and bulk of the transducer. The use of these patches would also be restricted to use on flat or single-curvature surfaces. The use of cardiac patches was thus abandoned. Proprietary conductive paint was investigated but was found to react with the piezoelectric paint, producing short circuits between the electrode and the substrate.

Vapour deposition of gold and aluminium is a well-established technique used in the preparation of electron microscope specimens. A metal vapour is produced inside a vacuum chamber by heating a bar of the metal to an appropriately high temperature. In the absence of any significant atmosphere, the vapour radiates away from the source until it encounters a body on which it condenses and deposits the metal. By this means a very thin, even layer of metal can be produced on an even surface, though for more complex surfaces care has to be taken to ensure line of sight between source and target.

The use of aluminium was not investigated because it would oxidize rapidly and so give rise to problems making electrical contact. The use of gold was found to be ideal. It adhered to the paint without difficulty with no tendency to delaminate, provided a very low-resistance layer with no electrical connection problems, worked well both during poling and as a transducer pickup and had negligible mass and stiffness owing to the extremely thin layer obtainable using this process. Finally and importantly, this use of high-purity gold is not expensive because it is only used in minute quantities. Vapour deposited gold was thus adopted for the electrode.

## 4 POLING CONDITIONS

The final stage in the manufacture of a transducer is to align the piezoelectric particles electrically by ‘poling’, i.e. by applying a large electrical potential to the electrodes to impose a strong electric field across the paint layer. The field strength and exposure time for poling had to be established by experiment. It was found that a poling potential of 600 V applied for 15 min across paint films of all PZT concentrations gave measurable voltage outputs at moderate strains. However, it was found that the output sensitivity diminished with time. This is a well-known phenomenon [7] which is conventionally overcome by poling for a sufficiently long period. It was found by experiment that poling for 1 h at 600 V gave an effect that was stable over an extended period. Poling for 1 h at 600 V was adopted as the standard procedure for all the transducers described in this paper.

## 5 MANUFACTURE OF SPECIMENS

The tests were carried out on short steel beams measuring $200 \times 40 \times 1.25$ mm with a rectangular sensor deposited on one face near the free end as shown in Fig. 1. Five batches of specimens were made, differing only in the concentration of PZT powder in the paint. The PZT concentrations were:

| | |
|---|---|
| Batch A | 75 wt % |
| Batch B | 75 wt % |
| Batch C | 60 wt % |
| Batch D | 45 wt % |
| Batch E | 80 wt % |

Batches A and B were made with nominally identical paints made up as separate batches on two occasions. This was done to test the variability of the process.

The transducers were made using the following procedure in all cases except for batch E. In this case the paint, with its very high solids concentration, could not be sprayed and the paint was applied by brush. In all other respects the transducer manufacturing process was identical:

1. The surface was cleaned using a commercial degreasing agent.
2. Most of the surface was masked off, leaving an area $50 \times 35$ mm exposed in the position required for the transducer.
3. A batch of paint was mixed in the proportion of resin to PZT required for the specimen.
4. The paint was sprayed on to the surface, care being taken to ensure even coverage, and allowed to cure at room temperature.
5. The edges of the dry paint were masked off, leaving

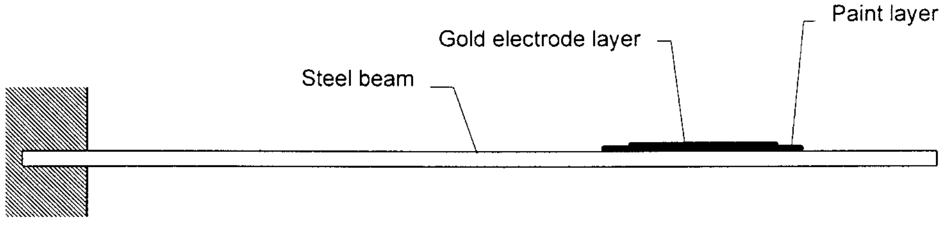

**Fig. 1** Test specimens

an area 40 × 25 mm exposed to form the final size of the transducer.

6. The beam was placed in the vacuum chamber and the gold layer deposited on to the surface.
7. The masking was then removed, leaving the transducer exposed on the surface of the beam, surrounded by a rim of exposed paint to ensure no contact between the gold electrode and the beam.
8. The substrate and electrode were connected to a constant voltage source and the paint film was poled at 600 V for 1 h.

## 6 TESTING FOR VARIATION BETWEEN TRANSDUCERS

Having established a technique for manufacturing piezoelectric paint strain transducers, it was necessary to investigate their effectiveness. The first test series was designed to investigate the variability between nominally identical transducers both within a single batch and between two batches made at different times using apparently identical paints. The second tested the effect of PZT concentration in the paint. Thirty specimens were prepared in five batches, using paints with four different concentrations of PZT powder in the resin binder.

In view of the large number of tests required, a simple test procedure was devised by which the transducer outputs at various strains could be compared without the need to measure the actual strain causing it. For each test the beam was mounted as a cantilever in a heavy block fixed to a bench. The beam was bent by deflecting the tip a given distance and then released and allowed to vibrate freely. The voltage output of the sensor was monitored directly using a digital oscilloscope.

As shown in the specimen trace (Fig. 2), the transducer signal does not exhibit a simple decaying sinusoidal waveform as might superficially be expected for a freely vibrating system excited with an initial displacement. There is an initial transient followed by a decaying periodic waveform. This is not an artefact of the measurement system, but is a real effect due to the difference between the deformed shape taken up by the beam under single-point end loading and the first resonance mode shape that the beam takes up after release. This difference excites higher vibration modes which quickly damp out.

As shown in the Appendix, the amplitudes of the periodic components of both the underlying damped sinusoidal first mode vibration and the bending strain at a point on the surface of the beam after any given number of cycles are proportional to the initial tip displacement once the transient components of the vibration have damped out. For comparative purposes, it is thus sufficient to measure the amplitudes of the piezoelectric transducer signals four cycles after release and relate this to the initial tip displacement to obtain a measure of the transducer sensitivity.

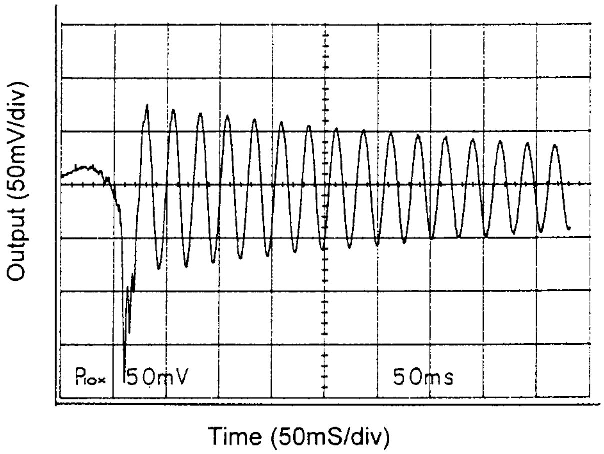

**Fig. 2** Typical waveform from the piezoelectric paint transducer monitoring the vibration of a cantilever beam released from rest with an initial static deformation due to a point load at its tip. The non-periodic transient portion of the response is due to high-frequency harmonics imposed by the difference in deformed shape of the beam under static and dynamic loading conditions (see the Appendix)

### 6.1 Tests for repeatability between batches

Figure 3 shows the scatter of output levels obtained from nominally identical specimens prepared in two separate batches, A and B, all prepared as described above using paint loaded with 75 per cent PZT powder by weight. From these results, comparisons of sensitivities can be obtained from the slopes of the outlying data points.

Two trends are clear from these results:

1. There is a great deal of scatter between the output sensitivities of the sensors. For batch A the maximum indicated sensitivity is approximately twice that of the minimum, and for batch B it is approximately three times greater.
2. The output sensitivities of transducers in batch B are generally greater than those of batch A, though there is some overlap between the populations.

### 6.2 Tests to assess the effect of the PZT concentration in the paint

Figure 4 shows the scatter of output levels obtained from transducers prepared from paints with PZT powder concentrations of 75, 60, 45 and 80 per cent by weight (batches B, C, D and E respectively). It is clear from these results that the low PZT concentration sensors in batches C and D are very much less sensitive than the higher concentration sensors in batches B and E.

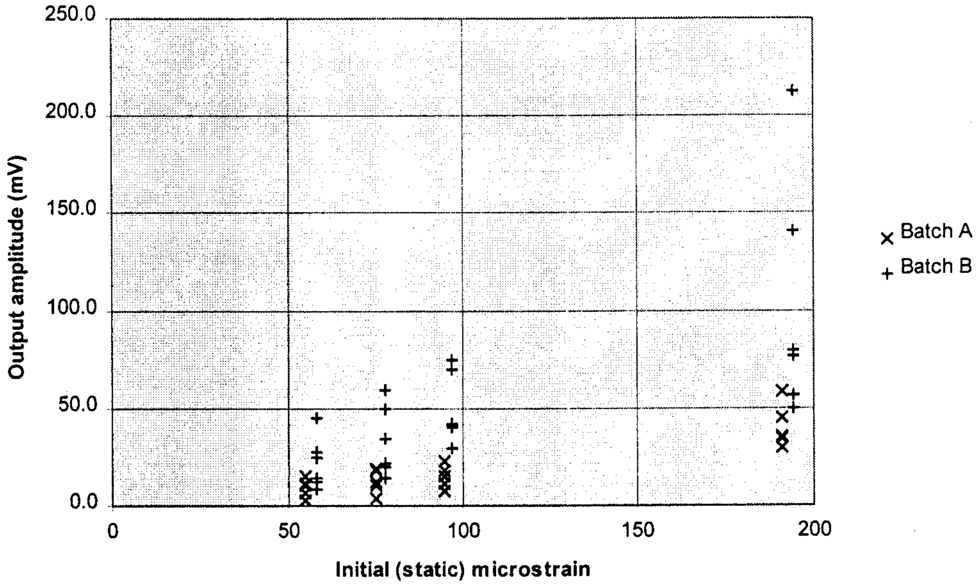

**Fig. 3** Scatter of output levels from two nominally identical batches of piezoelectric paint transducers, each containing 75 per cent PZT by weight

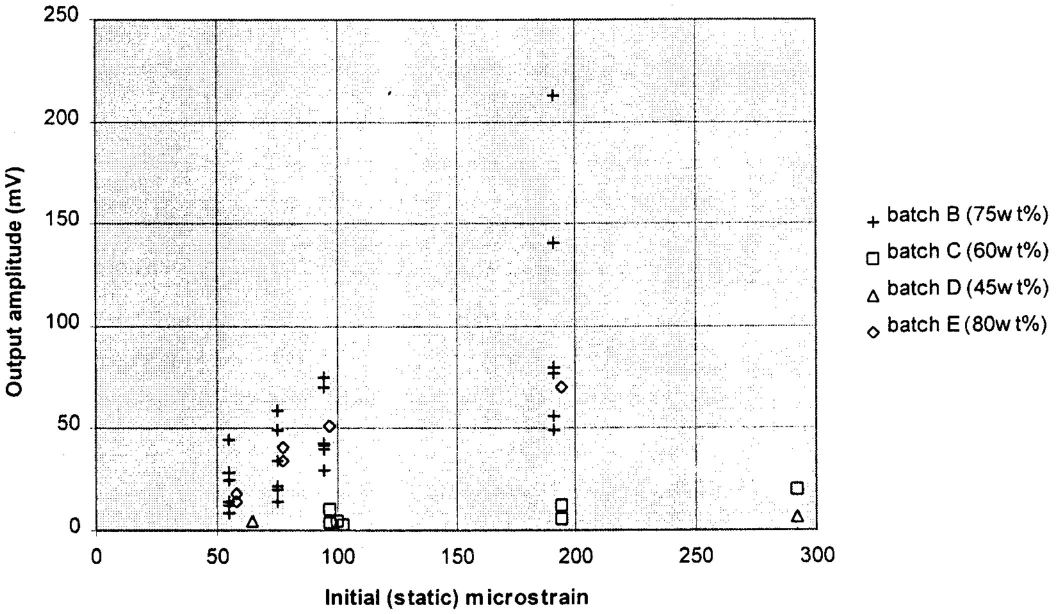

**Fig. 4** Scatter of output levels from piezoelectric paint transducers containing various concentrations of piezoelectric powder

However, the increase in PZT concentration from 75 to 80 per cent by weight has no clear effect, probably owing to the wide scatter of results in batch B.

It is clear from these results that a PZT concentration of 60 per cent by weight or less gives poor results, yielding very insensitive sensors (i.e. sensors that provide only a small signal for a given strain), but that 75 per cent by weight can give good sensitivity, albeit with wide variation between samples. Conversely, there is no evidence that a PZT concentration greater than 75 per cent by weight gives any improvement in sensitivity.

The most likely cause for the scatter of results is variation in the thickness of the paint film. It is demonstrated clearly by Egusa and Iwasawa [**2**] that the piezoelectric activity of PZT paint is related to the field intensity (voltage per unit distance) under which it is poled. Although the poling potential was held constant at 600 V for all the transducers used in this work, the thickness of the hand sprayed paint film would have varied considerably, both between specimens and from point to point within an individual transducer. This will have caused a significant variation in the poling field intensities which could easily account for the observed variation in transducer sensitivities. In view of the difficulty of applying 80 per cent by weight paint, which could only be done by brushing, compared with the spray application of 75 per cent by weight paint, and the lack of any evidence of improvement at the higher concentration, sensors made with 75 per cent by weight paint were taken forward for more detailed dynamic testing.

## 7 DETAILED DYNAMIC TESTING

More detailed dynamic tests were carried out on three specimens, all made with paint containing 75 per cent PZT but taken from batches A and B to obtain a representative spread of results. Full tests for dynamic range and bandwidth testing require measurements to be taken with the sensor excited sinusoidally over a range of frequencies and amplitudes in the steady state. To achieve this the specimens were mounted on an electromagnetic vibrator as shown in Fig. 5.

In order to relate the sensor signal to true strain, a strain gauge was mounted on the reverse side of the beam and connected to a commercial carrier amplifier (Fylde 641CA) in quarter bridge configuration. This arrangement gave adequate accuracy for this application since temperature compensation is not required for vibration amplitude measurements.

In the initial series of tests above, the output of the piezoelectric transducer was measured simply as a voltage by means of a digital oscilloscope. Piezoelectric transducers are more sensitive when charge, rather than voltage, is measured. In order to enable the sensors to work down to the very low strain levels required for a good estimate of dynamic range, charge was monitored using a commercial charge amplifier (Kistler 5001), the voltage output of which was in turn measured using a digital oscilloscope.

In each case the limits of the tests were defined by the piezoelectric signal dropping to the noise threshold, except for the high end of the dynamic range test which was defined by the maximum power available to drive the vibrator. The bandwidths and dynamic ranges of the three specimen transducers are shown in Figs 6 to 8. The signal level in each case is the ratio of the piezoelectric signal to the strain gauge signal (i.e. piezoelectric output per unit strain). These levels are presented in dB relative to a value near the middle of the working range.

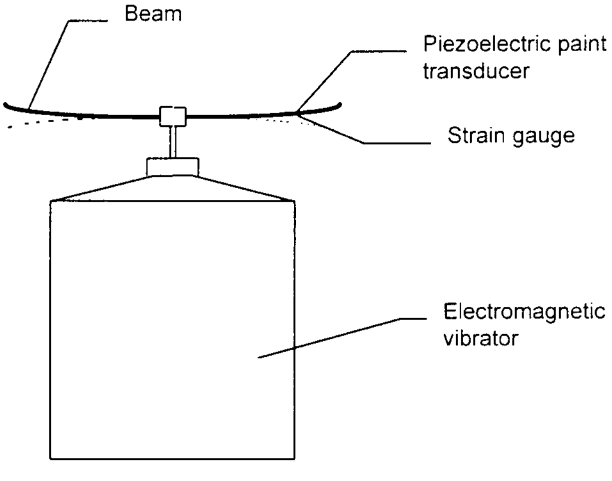

**Fig. 5** Dynamic testing arrangement

It may be seen that the form of the response curves is very similar for each transducer. The response curves over the measured dynamic range of two decades was quite flat in all three cases, never deviating from the nominal value by more than 1.5 dB. Apart from one specimen (B5) at low strains, all the responses were within ±0.5 dB of the reference sensitivities over the full two decade range. The frequency response is characterized in each case by a flat region of about one and a half decades centred on 100 Hz, with a rapid rise in sensitivity at the high and low frequency extremes. The ±3 dB ranges are approximately 20–600, 15–600 and 30–650 Hz respectively.

### 7.1 Analysis of bandwidth results

The reasons for the increase in sensitivity at high and low frequencies is not clear. However, it is evidently a real phenomenon and not an artefact of the experimental process.

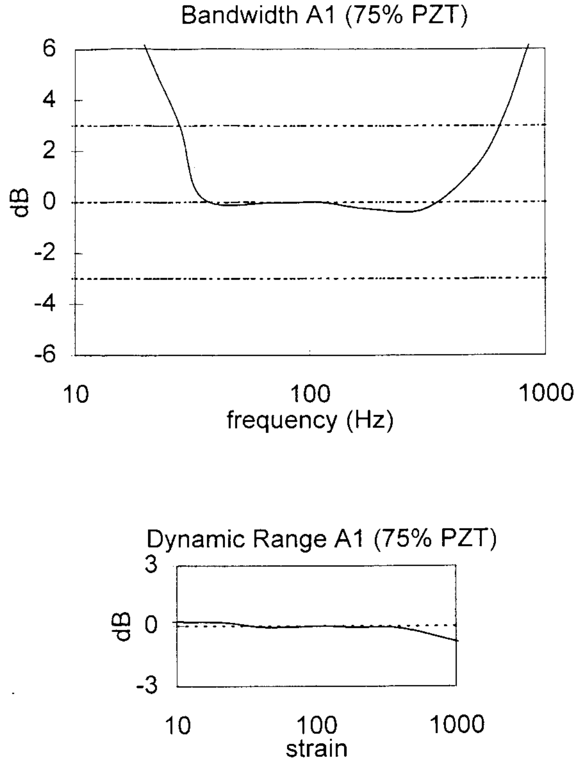

**Fig. 6** Bandwidth and dynamic range measurements for specimen A1 with 75 per cent concentration of PZT powder

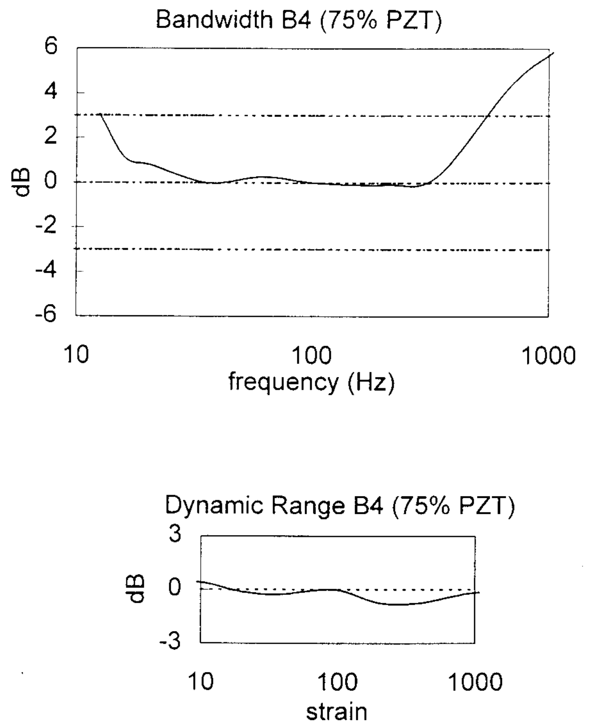

**Fig. 7** Bandwidth and dynamic range measurements for specimen B4 with 75 per cent concentration of PZT powder

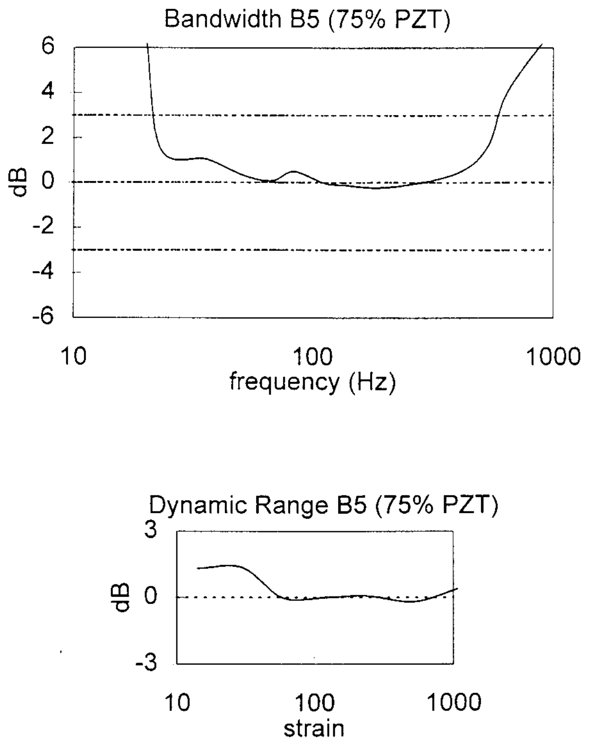

**Fig. 8** Bandwidth and dynamic range measurements for specimen B5 with 75 per cent concentration of PZT powder

The results described by Egusa and Iwasawa [**2**] show an unexplained rise in sensitivity at low frequencies, followed by a flat region up to their maximum frequency of 250 Hz. This is in good agreement with the results presented here. The high-frequency rise occurs at around 600 Hz. This is approaching the second mode resonance of the beam at about 800 Hz, and it was suggested that this could have influenced the response of the relatively large piezoelectric paint transducer. The strain varies significantly along its length in the higher vibration modes, whereas the strain gauge measures strain effectively at a point.

The mode shape for a vibrating beam of length $l$ is given by the expression [**8**]

$$y(x) = A\cosh(\beta x) + B\sinh(\beta x) + C\cos(\beta x) + D\sin(\beta x) \qquad (1)$$

where $A$, $B$, $C$ and $D$ are constants and $\beta$ is a value depending on the vibration mode and beam length. For the case of a cantilever beam in the second mode $(\beta l)^2 = 22.0$ [**8**]. The beam length $l$ is 90 mm, so $\beta = 52.12$. Equation (1) then reduces to

$$y(x) = A\{\cosh(\beta x) - \cos(\beta x) + 1.017[\sin(\beta x) - \sinh(\beta x)]\} \qquad (2)$$

i.e. the product of a constant $A$ defining the vibration amplitude and a function defining the mode shape.

From elementary beam theory, the bending strain at the surface is proportional to the second derivative:

$$\varepsilon(x) = \frac{t}{2}\frac{\mathrm{d}^2 y(x)}{\mathrm{d}x^2} = \frac{A\beta^2 t}{2}\{\cosh(\beta x) + \cos(\beta x) - 1.017[\sinh(\beta x) + \sin(\beta x)]\} \qquad (3)$$

where $t$ is the beam thickness.

The strain at the strain gauge position ($x = 55$ mm) is $-1.34K$, where $K$ is the constant value $(A\beta^2 t)/2$. The mean strain calculated from equation (3) over the transducer length $(0.35 < x < 0.75)$ is $-1.14K$. Therefore, the strain observed by the piezoelectric paint transducer, even at resonance, would not have differed from that observed by the strain gauge by more than 20 per cent. Evidently then, the observed limits to the 'flat' bandwidth are real.

It should be noted that the three specimens tested were selected to be representative of the two batches made with 75 per cent loading of PZT powder and to cover the fairly wide range of sensitivities indicated in the earlier statistical testing. In fact specimens A1 and B5 had a fairly similar sensitivity, but B4 was approximately three times more sensitive. It is clear that this absolute variation between the specimens is not reflected in any variation in their dynamic responses, indicating that a piezoelectric paint transducer could be expected to give reliable measurements within the frequency range 30–600 Hz provided it is individually calibrated at a single frequency and strain amplitude.

## 8 CONCLUSIONS

1. Materials and fabrication techniques have been identified for the manufacture of piezoelectric paint-based dynamic strain sensors, though these have not been optimized.
2. There is considerable variation between nominally identical transducers made by these techniques, probably owing to variations in thickness of the paint layer.
3. The dynamic range for the transducers is good, with no more than 1.5 dB deviation from a flat response over two decades of strain magnitude.
4. The bandwidth for the transducers is less good, with a ±3 dB bandwidth of less than one and a half decades of frequency. However, the useful range is between 30 and 600 Hz, which is the most important range for many structural vibration applications.
5. The concept of a strain transducer based on piezoelectric paint clearly works. However, the variation between transducers as currently made necessitates individual calibration. Further development is required to make the fabrication process more repeatable.

## REFERENCES

1 **White, N. M.** and **Turner, J. D.** Thick film sensors: past, present and future. *Measmt Sci. Technol.*, 1997, **8**(1), 1–20.

2 **Egusa, S.** and **Iwasawa, N.** Piezoelectric paints: preparation and application as built-in vibration sensors of structural materials. *J. Mater. Sci.*, March 1993, **28**(6), 1667–1672.

3 **Egusa, S.** and **Iwasawa, N.** Application of piezoelectric paints to damage detection in structural materials. *J. Reinf. Plast. and Composites*, August 1996, **15**(8), 806–817.

4 **Lukacs, M., Sayer, M., Knapik, D., Candela, R.** and **Foster, F. S.** Novel PZT films for ultrasound biomicroscopy. In Proceedings of IEEE Ultrasonics Symposium, 1996, Vol. 2, pp. 901–904.

5 **Turner, G. P. A.** *Introduction to Paint Chemistry and Principles of Paint Technology*, 3rd edition, 1988 (Chapman and Hall, London).

6 **Hanner, K. A., Safari, A., Newnham, R. E.** and **Runt, J.** Thin film 0–3 polymer piezoelectric ceramic composites—piezoelectric paints. *Ferroelectrics*, 1989, **100**, 255–260.

7 **Moulsdon, A. J.** and **Herbert, J. M.** *Electroceramics*, 1992 (Chapman and Hall, London).

8 **Thompson, W. P.** *Theory of Vibration with Applications*, 1983 (George Allen and Unwin, London).

## APPENDIX

Strain energy in the statically deformed beam (see Fig. 9) is given by

$$U=\frac{1}{2}\int_0^l\int_{-t/2}^{t/2}\sigma(a,x)\varepsilon(a,x)\,\mathrm{d}a\,\mathrm{d}x$$

$$=\frac{E}{2}\int_0^l\int_{-t/2}^{t/2}\varepsilon^2(a,x)\,\mathrm{d}a\,\mathrm{d}x \qquad (4)$$

where

$x$ = position along the beam
$a$ = distance from the neutral axis
$\sigma$ = local stress
$\varepsilon$ = local strain
$t$ = beam thickness
$l$ = beam length

Taking the standard form for the bending moment in a statically deformed cantilever beam with tip deflection $Y$:

$$M(x)=\frac{3EIY}{l^3}(l-x) \qquad (5)$$

Using elementary beam theory, the bending stress at the surface of the beam is given by

$$\sigma(x)=\frac{M(x)}{I}\frac{t}{2}$$

and therefore

$$\varepsilon(x)=\frac{M(x)}{EI}\frac{t}{2} \qquad (6)$$

Substituting equation (5) in equation (6):

$$\varepsilon(x)=\frac{3t(l-x)}{2l^3}Y$$

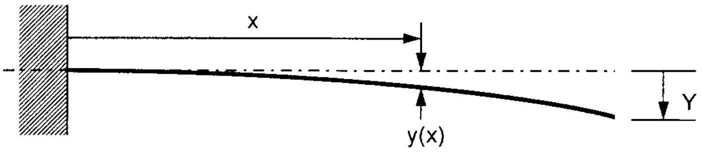

**Fig. 9** Cantilever beam deformed statically by a point load at the tip or in first mode free vibration. Note that the deformed shape is slightly different for the two cases, but the difference is omitted for clarity

i.e. the surface strain at any point on the beam is proportional to the initial tip deflection $Y$. Hence, strain energy

$$U \propto Y^2 \tag{7}$$

The kinetic energy in the freely vibrating beam is given by

$$T = \frac{1}{2}\int_0^l \rho \dot{y}^2(x)\,\mathrm{d}x$$

where

$\rho$ = mass per unit length of the beam
$\dot{y}$ = local particle velocity

Assuming sinusoidal motion at all points on the beam $y(x) = \hat{y}(x)\sin(\omega t)$, the peak kinetic energy is given by

$$\hat{T} = \frac{\omega^2 \rho}{2}\int_0^l \rho \hat{y}^2(x)\,\mathrm{d}x$$

where

$y$ = local displacement of the beam
$\hat{y}$ = local vibration amplitude
$\omega$ = vibration frequency (rad/s)

As shown in equation (2), the vibration amplitude at any point is proportional to some constant $A$. That is, the mode shape is independent of the amplitude and so, at any given frequency, the amplitude at the general point $\hat{y}(x)$ is proportional to the amplitude at any specific point, say $\hat{y}(l)$, the amplitude at the beam tip. Therefore,

$$\hat{T} \propto \hat{y}^2(l) \tag{8}$$

Equating peak kinetic energy and initial strain energy from equations (7) and (8), $\hat{y}^2(l) \propto Y^2$, so that $\hat{y}(l) \propto Y$ and hence $\hat{y}(x) \propto Y$, i.e. the vibration amplitude throughout the beam is proportional to the initial static tip displacement.

As shown in equation (3), the peak strain at any point $\hat{\varepsilon}(x)$ is also proportional to the constant $A$, so by the same argument is proportional to the initial static tip displacement $Y$. It is thus legitimate to use the initial static tip deflection as a measure of strain amplitude in the vibrating beam for the comparative tests described in this report.